N-64-8

NWL Report No. 1887

GENERAL LAMINAR FLOWS PAST LEADING AND TRAILING EDGES OF A SEMI-INFINITE PLATE

by

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Task Assignment
No. R360FR103/2101/R01101001

October 1963

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ABSTRACT

The characteristics of general laminar flows around the edge of a semi-infinite flat plate are investigated on the basis of the complete Navier-Stokes equations. The solution of this singular and nonlinear problem is obtained in an exact and linear manner in the form of a double series around the edge. It permits a rigorous discussion of the properties of flows around an edge, which are of importance in the practical applications as well as in the theory of viscous and non-viscous fluid motions. In particular, it becomes possible to check the reliability of the Kutta condition which represents a fundamental proposition in ideal flow theory.

FOREWORD

This work was sponsored by the Naval Weapons Laboratory Foundational Research Program. It was performed in the Computation and Analysis Laboratory as part of the Meteorological and Oceanographical project (R360FR103/2101/R01101001).

The authors wish to extend their thanks to Dr. C. J. Cohen for many critical and stimulating discussions.

The date of completion was 4 October 1963.

Approved for Release:

/s/ R. H. LYDDANE
Technical Director

1. Introduction

The flows around sharp edges can be described in a realistic way only if friction effects are included. For the mathematical treatment of this problem the complete Navier-Stokes equations must be used. Early attempts to solve the problem on the basis of ideal flow theory by the introduction of discrete vortices or continuous vortex sheets, are in principle of hypothetical nature and must be confirmed, a posteriori, by the real flow behavior. It turned out that the boundaries of two-phase flows (for instance, cavities) could be described quite well, whereas the approach for one-phase flows (for instance, wakes) is not satisfactory (see [2]).

In a recent paper [7] a method has been presented to solve the edge problem with the Navier-Stokes equations for the special case of symmetric flows past a semi-infinite plate. The same procedure will be applied in this paper to discuss the general nonsymmetric flows in the vicinity of the edge. Though the discussion is essentially qualitative, valuable information about the laminar flow characteristics are obtained. They reveal not only patterns of flows towards or from an oblique plate and explain phenomena observed in photographs, but they permit also a reexamination of propositions in ideal flow theory. For example, it can be shown that separation at the edge occurs, which contradicts the well-known Kutta condition.

2. Reduction of the Equations of Motion

For a two-dimensional steady-state flow the Navier-Stokes equations and the equation of continuity are in a Cartesian coordinate system (x, y)

$$uu_x + vu_y = -\frac{1}{\rho} p_x + \nu(u_{xx} + u_{yy}) \tag{1}$$

$$uv_x + vv_y = -\frac{1}{\rho} p_y + \nu(v_{xx} + v_{yy}) \tag{2}$$

$$u_x + v_y = 0 \tag{3}$$

where (u, v) is the velocity, p the pressure, ρ the constant density, and ν the constant kinematic viscosity of the fluid.

Considering the flow past a semi-infinite flat plate, which is located along the positive x-axis, one has to satisfy the nonslip condition at the plate

$$\left.\begin{matrix} x \geq 0 \\ y = \pm 0 \end{matrix}\right\} \quad u = 0, \quad v = 0 \tag{4}$$

According to [7] it is useful to remove the singularity at the origin by the introduction of parabolic coordinates, which are made dimensionless by a constant length σ^2. This may be chosen in such a way that its value becomes characteristic for the flows under consideration (see [7]). The transformation is

$$x = \sigma^2(\xi^2 - \eta^2) \tag{5}$$

$$y = 2\sigma^2 \xi\eta \quad . \tag{6}$$

The velocity and the pressure are also replaced by dimensionless quantities

$$U = \frac{\sigma^2}{\nu} u \ , \qquad V = \frac{\sigma^2}{\nu} v \ , \qquad P = -\,2\,\frac{\sigma^4}{\nu^2}\,\frac{p}{\rho} \quad . \tag{7}$$

One obtains for the equations (1) through (3)

$$2(\xi U + \eta V)U_\xi + 2(\xi V - \eta U)U_\eta = -\,\eta P_\eta + \xi P_\xi + U_{\xi\xi} + U_{\eta\eta} \tag{8}$$

$$2(\xi U + \eta V)V_\xi + 2(\xi V - \eta U)V_\eta = \eta P_\xi + \xi P_\eta + V_{\xi\xi} + V_{\eta\eta} \tag{9}$$

$$\xi(U_\xi + V_\eta) + \eta(V_\xi - U_\eta) = 0 \tag{10}$$

with the nonslip condition

$$\eta = 0 \quad : \qquad U = 0, \qquad V = 0 \tag{11}$$

at the plate. The analytic properties of the solutions permit Taylor expansions of the integrals around $\xi = 0$:

$$U = \sum_{n=0}^{\infty} U_n(\eta)\xi^n \quad , V = \sum_{n=0}^{\infty} V_n(\eta)\xi^n \quad , P = \sum_{n=0}^{\infty} P_n(\eta)\xi^n \tag{12}$$

With these series the partial differential equations (8) through (10) are reduced to the infinite set of ordinary differential equations

$$U_n'' + (n+1)(n+2)U_{n+2} - \eta P_n' + nP_n =$$

$$2\sum_{i=0}^{n+1} iU_i(U_{n-i} + \eta V_{n-i+1}) + 2\sum_{i=0}^{n} U_i'(V_{n-i-1} - \eta U_{n-i}) \qquad (13)$$

$$V_n'' + (n+1)(n+2)V_{n+2} + (n+1)\eta P_{n+1} + P_{n-1}' =$$

$$2\sum_{i=0}^{n+1} iV_i(U_{n-i} + \eta V_{n-i+1}) + 2\sum_{i=0}^{n} V_i'(V_{n-i-1} - \eta U_{n-i}) \qquad (14)$$

$$nU_n + (n+1)\eta V_{n+1} - \eta U_n' + V_{n-1}' = 0 \qquad (15)$$

for $n = 0, 1, 2 \ldots$. The primes denote the derivatives with respect to η. The nonslip condition (11) requires

$$\eta = 0: \quad U_n = 0 \,, \quad V_n = 0 \,. \qquad (16)$$

For the sake of simplicity in the forthcoming derivations the first 19 equations of the system (13) through (15) are listed in Appendix A.

The coefficient functions U_n, V_n, and P_n are assumed in the form of series expansions

$$U_n = \sum_{m=1}^{\infty} U_{nm}\eta^m \,, \qquad V_n = \sum_{m=2}^{\infty} V_{nm}\eta^m \,, \qquad P_n = \sum_{m=0}^{\infty} P_{nm}\eta^m \qquad (17)$$

in which the nonslip condition (16) and the condition

$$V_{n1} = 0 \text{ for all } n = 0, 1, 2 \ldots \qquad (18)$$

have been considered already. The latter one follows immediately from the reduced continuity equations (15). As was pointed out in [7] the existence of an integral for the set (13) through (15) is not certain since the differential equations are singular at the origin $\eta = 0$. The existence proof is achieved in the same way as in [7] by substituting the series (17) into the differential equations (13), (14), and (15). These reduce to an infinite set of algebraic equations, which consists of an infinite number of finite subsets, all of which are linear in the unknown coefficients. The number of free parameters for each linearly dependent subset is now two instead of one parameter found in the special case of symmetric flows. They are designated by A_n and B_n in such a way that all B_n vanish for symmetric flows. Therefore, the family of all symmetric flows may be defined by the condition

$$B_n = 0 \qquad (n = 2, 3, 4, \ldots) \, . \tag{19}$$

The subsets of algebraic equations are found by collecting the algebraic equations of the same order, where the order is defined by the sum of the exponents of η and ξ. The first subset yields vanishing coefficients because of the conditions (16) and (18). The second subset follows from

$$(A - 1)\eta^1 : \qquad U_{02} = 0 \tag{20}$$

$$(A - 2)\eta^1 : \quad U_{11} - U_{11} = 0 \tag{21}$$

$$(A - 9)\eta^0 : \quad V_{02} = 0 \tag{22}$$

This system has the solution

$$U_{02} = 0 , \quad U_{11} = B_2 , \quad V_{02} = 0 \tag{23}$$

where B_2 is the free parameter. The pressure is normalized by assuming a zero pressure at the edge of the plate:

$$P_{00} = 0 \tag{24}$$

The next subsets up to the seventh order are recorded in Appendix B. Their two-parameter solutions are:

Third order subset (with A_3 and B_3 as parameters):

$$\begin{array}{lll} U_{03} = A_3 & V_{03} = B_3 & P_{01} = -6A_3 \\ U_{12} = 3B_3 & V_{12} = 3A_3 & P_{10} = -6B_3 \\ U_{21} = -6A_3 & & \end{array} \tag{25}$$

Fourth order subset (with A_4 and B_4 as parameters):

$$\begin{array}{lll} U_{04} = 0 & V_{04} = 0 & P_{02} = A_4 \\ U_{13} = B_4 & V_{13} = 0 & P_{11} = -2B_4 \\ U_{22} = A_4 & V_{22} = B_4 & P_{20} = -A_4 \\ U_{31} = -B_4 & & \end{array} \tag{26}$$

Fifth order subset (with A_5 and B_5 as parameters):

$U_{05} = A_5$	$V_{05} = B_5$	$P_{03} = -10A_5$	
$U_{14} = 5B_5$	$V_{14} = 5A_5$	$P_{12} = -30B_5$	
$U_{23} = -25A_5$	$V_{23} = 5B_5$	$P_{21} = 30A$	(27)
$U_{32} = -15B_5$	$V_{32} = -15A_5$	$P_{30} = 10B_5$	
$U_{41} = 10A_5$			

Sixth order subset (with A_6 and B_6 as parameters):

$U_{06} = 0$	$V_{06} = 0$	$P_{04} = 3A_6$	
$U_{15} = B_6$	$V_{15} = 0$	$P_{13} = -8B_6$	
$U_{24} = 6A_6$	$V_{24} = 4B_6$	$P_{22} = -18A_6$	(28)
$U_{33} = -6B_6$	$V_{33} = 4A_6$	$P_{31} = 8B_6$	
$U_{42} = -6A_6$	$V_{42} = -4B_6$	$P_{40} = 3A_6$	
$U_{51} = 2B_6$			

Seventh order subset (with A_7 and B_7 as parameters):

$$7U_{07} = A_7 \qquad V_{07} = B_7$$

$$U_{16} = 7B_7 - \frac{1}{5}B_2A_3 \qquad V_{16} = A_7$$

$$U_{25} = -8A_7 - \frac{1}{7}B_2B_3 \qquad V_{25} = 14B_7 - \frac{1}{2}B_2A_3$$

$$U_{34} = -70B_7 + \frac{3}{2}B_2A_3 \qquad V_{34} = -10A_7 - \frac{1}{7}B_2B_3$$

$$U_{43} = 15A_7 + \frac{3}{14}B_2B_3 \qquad V_{43} = -35B_7 + B_2A_3$$

$$U_{52} = 35B_7 - B_2A_3 \qquad V_{52} = 5A_7 + \frac{1}{14}B_2B_3$$

$$U_{61} = -2A_7 - \frac{1}{35}B_2B_3$$

$$
\begin{aligned}
P_{05} &= -2A_7 - \frac{4}{35}B_2B_3 \\
P_{14} &= -70B_7 + B_2A_3 \\
P_{23} &= 20A_7 + \frac{2}{7}B_2B_3 \\
P_{32} &= 140B_7 - 4B_2A_3 \\
P_{41} &= -10A_7 - \frac{1}{7}B_2B_3 \\
P_{50} &= -14B_7 + \frac{2}{5}B_2A_3
\end{aligned}
\tag{29}
$$

With the equations (12), (17) and the coefficients (23) through (29) one obtains for the velocity components U, V and the pressure P the double series

$$
\begin{aligned}
U = {} & B_2 \eta\xi \\
& - 6A_3 \eta\xi^2 + 3B_3 \eta^2\xi + A_3 \eta^3 \\
& - B_4 \eta\xi^3 + A_4 \eta^2\xi^2 + B_4 \eta^3\xi \\
& + 10A_5 \eta\xi^4 - 15B_5 \eta^2\xi^3 - 25A_5 \eta^3\xi^2 + 5B_5 \eta^4\xi + A_5 \eta^5 \\
& + 2B_6 \eta\xi^5 - 6A_6 \eta^2\xi^4 - 6B_6 \eta^3\xi^3 + 6A_6 \eta^4\xi^2 + B_6 \eta^5\xi \\
& - \left(2A_7 + \frac{1}{35}B_2B_3\right)\eta\xi^6 + (35B_7 - B_2A_3)\eta^2\xi^5 + \left(15A_7 + \frac{3}{14}B_2B_3\right)\eta^3\xi^4 \\
& - \left(70B_7 - \frac{3}{2}B_2A_3\right)\eta^4\xi^3 - \left(8A_7 + \frac{1}{7}B_2B_3\right)\eta^5\xi^2 + \left(7B_7 - \frac{1}{5}B_2A_3\right)\eta^6\xi \\
& + \frac{1}{7}A_7 \eta^7
\end{aligned} \tag{30}
$$

$$
\begin{aligned}
V = {} & 3A_3 \eta^2\xi + B_3 \eta^3 \\
& - B_4 \eta^2\xi^2 \\
& - 15A_5 \eta^2\xi^3 + 5B_5 \eta^3\xi^2 + 5A_5 \eta^4\xi + B_5 \eta^5 \\
& - 4B_6 \eta^2\xi^4 + 4A_6 \eta^3\xi^3 + 4B_6 \eta^4\xi^2 \\
& + \left(5A_7 + \frac{1}{14}B_2B_3\right)\eta^2\xi^5 - (35B_7 - B_2A_3)\eta^3\xi^4 - \left(10A_7 + \frac{1}{7}B_2B_3\right)\eta^4\xi^3 \\
& + \left(14B_7 - \frac{1}{2}B_2A_3\right)\eta^5\xi^2 + A_7 \eta^6\xi + B_7 \eta^7
\end{aligned} \tag{31}
$$

$$
\begin{aligned}
P = {} & - 6B_3 \xi - 6A_3 \eta \\
& - A_4 \xi^2 - 2B_4 \eta\xi + A_4 \eta^2 \\
& + 10B_5 \xi^3 + 30A_5 \eta\xi^2 - 30B_5 \eta^2\xi - 10A_5 \eta^3 \\
& + 3A_6 \xi^4 + 8B_6 \eta\xi^3 - 18A_6 \eta^2\xi^2 - 8B_6 \eta^3\xi + 3A_6 \eta^4 \\
& - \left(14B_7 - \frac{2}{5}B_2A_3\right)\xi^5 - \left(10A_7 + \frac{1}{7}B_2B_3\right)\eta\xi^4 + (140B_7 - 4B_2A_3)\eta^2\xi^3 \\
& + \left(20A_7 + \frac{2}{7}B_2B_3\right)\eta^3\xi^2 - (70B_7 - B_2A_3)\eta^4\xi - \left(2A_7 - \frac{4}{35}B_2B_3\right)\eta^5
\end{aligned} \tag{32}
$$

The description of the flow behavior by the first terms of the series expansions (30), (31), and (32) is valid in the neighborhood of the edge. It is significant that in these double series nonlinear inertial terms do not appear before the seventh order. Therefore, the flow region near the edge behaves like a flow field of Stokes' type. This is actually the key to the understanding of the detailed description of the flow properties without knowing the asymptotic flow far away from the edge. As was pointed out in [6] and [7], every flow forms a "slow-motion layer" at a rigid body, for which the inertial forces may be neglected. However, the inertial forces become dominant outside the slow-motion layer and determine the asymptotic ideal flows. For this reason, a slow-motion flow is in principle not possible, if it is required to have asymptotic ideal flow properties.

In this connection a special case of the general solutions (30), (31), and (32) may be anticipated, which belongs to the family of the Couette flows and which is characterized by the property that the plate does not disturb the surrounding flow. Assuming all coefficients to be zero except B_2 and A_4 one obtains

$$U = B_2 \eta \xi + A_4 \eta^2 \xi^2 \tag{33}$$

$$V \equiv 0 \tag{34}$$

$$P = A_4 (\eta^2 - \xi^2) \tag{35}$$

or

$$u = \frac{\nu}{\sigma^2}\left(\frac{B_2}{2\sigma^2}y + \frac{A_4}{4\sigma^4}y^2\right) \qquad (36)$$

$$v = 0 \qquad (37)$$

$$\frac{p}{\rho} = \frac{\nu^2}{2\sigma^4}\frac{A_4}{\sigma^2}x \quad . \qquad (38)$$

In contrast to the symmetric Couette flows, which are defined by $B_2 = 0$, the nonsymmetric flows (36), (37), and (38) cause a shear stress at the plate of the magnitude

$$\nu\rho u_y(x, 0) = \nu^2\rho\frac{B_2}{2\sigma^4} \quad , \qquad (39)$$

which has a positive sign at the upper side of the plate and a negative sign at the lower side, so that they balance each other and yield no resistance.

It might be mentioned that the existence of the slow motion layer at a rigid body is one reason why Prandtl's boundary layer theory cannot describe the "thin friction layer" at the body in an exact manner since the friction terms in the Navier-Stokes equations are truncated (see [6], [7]).

3. Discussion of Flow Properties

Each combination of the free parameters A_n and B_n represents an exact solution of the Navier-Stokes equations. From the infinite number of possibilities, which reflects the generality of the Navier-Stokes equations, only simple cases of practical interest may be pointed out. These selected examples may be divided into attached and separated flows around the edge.

The distinction between these two classes follows from the study of the streamlines at the edge. If no streamlines originate at the edge, the flow may be called attached; otherwise the flow is separated. The latter class includes also the trivial cases such as the parallel flows toward or away from the plate (Figures 1a and 2a).

The streamlines at the edge are described by the differential equation

$$\frac{dy}{dx} = \frac{\eta d\xi + \xi d\eta}{\xi d\xi - \eta d\eta} = \frac{v}{u} = \frac{V}{U} \tag{40}$$

or

$$\frac{d\eta}{d\xi} = \frac{- B_2\eta\xi + A_3\eta(9\xi^2 - \eta^2) + \ldots}{B_2\xi^2 + A_3\xi(4\eta^2 - 6\xi^2) + \ldots} \tag{41}$$

In addition, it is useful to know the shear stress at the plate, which follows from equation (30)

$$\nu\rho u_y(x, 0) = \frac{\nu^2\rho}{2\sigma^4\xi} U_\eta(\xi, 0) = \frac{\nu^2\rho}{2\sigma^4}(B_2 - 6A_3\xi - B_4\xi^2 + 10A_5\xi^3 + \ldots) \tag{42}$$

The discussion of the flow properties may be guided by envisioning the following experiment: Let the motion be symmetrical around the semi-infinite plate, where the flow toward the plate is called stagnation flow past the (leading) edge and the flow away from the plate is designated as wake flow around the (trailing) edge (Figures 1a and 2a). Keeping the Reynolds number constant one may change the flow direction slowly up to a right angle of attack. One observes then the following flow behavior: Under a sufficiently small angle of attack a streamline will always end or begin at the edge exactly as in the symmetric case (Figures 1b and 2b). For larger angles of attack this streamline vanishes from the edge, and the fluid particles pass smoothly around the leading and trailing edges (attached flows in Figures 1c and d, 2c and d). The flow remains attached as long as the Reynolds number is sufficiently small. Otherwise, from a certain angle of attack on separation occurs (Figures 3 and 4).

From this description the following types of flows are distinguished:

a. Symmetric flows

They are discussed already in [7] and are defined by the condition

$$A_3 = 0,\ A_4A_5 > 0\ ,\ B_n = 0\ (n = 2, 3, \ldots) \qquad (43)$$

The flows around leading and trailing edges are determined by the positive and negative signs of the coefficient A_5. Condition (43) includes only symmetric motions of the trivially separated flow type.

b. Quasi-symmetric flows

In contrast to ideal flow theory, where an infinitesimal small angle of attack causes the disappearance of the dividing streamline from the edge, the real flow can stand a slight nonsymmetry without losing the properties of symmetric flows in the leading terms. The conditions are the same as in (43) except that a nonsymmetry arises from the seventh order on (Figures 1b and 2b):

$$A_3 = 0,\ A_4A_5 > 0.\ B_n = 0 \text{ for } n < 7,\ B_7 \neq 0\ . \qquad (44)$$

The fact that a streamline at the edge exists is anticipated and will be shown farther below.

Because of the same behavior of symmetric and quasi-symmetric flows the theorems established in [7] for symmetric flows are valid for both classes. Therefore, one obtains for the shear stress and the pressure the following extended theorems:

THEOREM 1: In symmetric and quasi-symmetric laminar flows past a semi-infinite flat plate the shear stress at the surface is zero at the edge and increases or decreases as the three-halves power of the distance from the edge.

THEOREM 2: In symmetric and quasi-symmetric laminar flows past a semi-infinite flat plate the pressure varies at the edge in a strictly linear manner.

c. <u>Almost skew-symmetric flows</u>

For larger angles of attack the streamline, which originates at the edge, vanishes, and the flow direction at the upper and lower side of the plate is opposite. The coefficient B_2 is no longer zero. However, one observes still a symmetric property which is caused by the coefficient B_4 in equation (31). This type of motion may, therefore, be called "almost skew-symmetric" flow and is described by the condition

$$A_3 = 0,\ A_5 > 0,\ B_2B_3 > 0,\ B_3B_4 > 0 \qquad (45)$$

for the leading edge and

$$A_3 = 0,\ A_5 < 0,\ B_2B_3 > 0,\ B_3B_4 < 0 \qquad (46)$$

for the trailing edge (Figures 1c and 2c).

The non-zero coefficient B_2 results in a change of the flow characteristics which become similar to the type of Couette flows (33). Indeed, the lowest order of the velocity component u is linear in the coordinate y. The shear stress at the plate is constant in the very neighborhood of the edge. This result differs from that for symmetric and quasi-symmetric flows formulated in Theorem 1.

The pressure drop at the plate is also different from that of the symmetric and quasi-symmetric flows (Theorem 2). As the coefficient B_3 is not zero the pressure becomes

$$p = \rho \frac{\nu^2}{2\sigma^4}\left(\pm 6 \frac{B_3}{\sigma} \sqrt{x} - \frac{A_4}{\sigma^2} x \pm \ldots \right) \quad (x \geq 0,\ y = 0) \tag{47}$$

Therefore, the pressure decreases near the edge in flow direction with the square root of the distance from the edge. This fact is surprising from the standpoint of ideal flow theory, where an extremum of the body contour effects an extremal pressure. A comparison of the pressure distribution around a circular cylinder for ideal and real flows shows on the other hand [5] that the expected extremal behavior is not necessarily true for real flows.

d. Distorted nonsymmetric flows

A further increase of the angle of attack leads to a type of flows which has already the tendency to separate (Figures 1d and 2d). This is due to the fact that the coefficient A_3 is no longer zero. One can easily see that the term $3A_3\eta^2\xi$ in equation (31) for the velocity component V distorts the symmetry.

The flows of this type are characterized by

$$A_3 \gtrless 0 \ , \quad B_2B_3 > 0 \tag{48}$$

for the leading and trailing edges, respectively.

The statements on shear stress and pressure drop at the plate for almost skew-symmetric flows are also valid for distorted nonsymmetric flows. Therefore general nonsymmetric flows have the following properties:

THEOREM 3: In almost skew-symmetric and distorted nonsymmetric laminar flows around the edge of a semi-infinite flat plate the shear stress at the surface is constant near the edge. The flow behaves like a Couette flow.

THEOREM 4: In almost skew-symmetric and distorted nonsymmetric laminar flows around the edge of a semi-infinite flat plate the pressure at the surface decreases in flow direction as the square root of the distance from the edge.

e. Separated flows

Distorted nonsymmetric flows around the edge can lead to separation from the edge. Separated flows of this type are characterized by the existence of nontrivial solutions of the differential equation (41). A solution is assumed by the series expansion

$$\xi = \alpha\eta + \beta\eta^2 + \gamma\eta^3 + \dots \ , \tag{49}$$

which reduces the differential equation (41) to an infinite set of algebraic equations. The first four of these equations are:

$$\eta^2 : \quad B_2\alpha^2 = 0 \tag{50}$$

$$\eta^3 : \quad 15A_3\alpha^3 - 5B_3\alpha^2 - 5A_3\alpha - B_3 = 0 \tag{51}$$

$$\eta^4 : \quad 3B_2\beta^2 + 12B_3\alpha\beta + 6(1 - 9\alpha^2)A_3\beta + 3(1 - \alpha^2)\alpha^2 B_4 + 2A_4\alpha^3 = 0 \tag{52}$$

$$\eta^5 : \quad 35A_5\alpha^5 - 35B_5\alpha^4 - 7(10A_5 + 2\beta B_4)\alpha^3 + 7(2B_5 - 9\gamma A_3 + \beta A_4)\alpha^2 + 7(2\gamma B_3 - 9A_3\beta^2 + A_5 + B_4\beta)\alpha + 7A_3\gamma + 7B_3\beta^2 + 7B_2\beta\gamma + B_5 = 0 \tag{53}$$

There follows immediately from equation (50) that a non-vanishing coefficient α requires the coefficient B_2 to be zero. In the physical (x, y) - plane the condition $\alpha \neq 0$ means that the plate is not tangential to this streamline. In this case motions which display Couette flow properties ($B_2 \neq 0$) are not possible.

It is easy to verify by means of the velocity components U and V that under the assumption $A_3B_3 \neq 0$ only the combination

$$A_3 > 0 \quad , \quad \frac{B_3}{\alpha} > 0 \tag{54}$$

is of practical interest. This class of flows is identified as separated flows around the leading edge of a plate (Figure 3).

The shear stress at the surface is now according to equation (42)

$$\nu\rho u_y(x, 0) = - 3 \frac{\nu^2\rho}{\sigma^4} A_3 \sqrt{x} - \ldots \tag{55}$$

on both sides of the plate.

THEOREM 5: In separated laminar flows around the leading edge of a semi-infinite flat plate, where the plate is not tangential to the separating streamline, the shear stress at the surface is zero at the edge and increases as the square root of the distance from the edge.

The pressure drop at the plate is given by equation (47) and is the same as for nonsymmetric attached flows. However, the flow direction is in the present case the same on both sides of the plate.

The coefficient α is determined by the cubic equation (51) which shows immediately the existence of a real root and which may be written in the form

$$3A_3(\alpha^2 - \frac{1}{3}) = \frac{B_3}{\alpha}(\alpha^2 + \frac{1}{5}) \quad . \tag{56}$$

Considering the restriction (54) one realizes the condition

$$\alpha^2 > \frac{1}{3} \tag{57}$$

which results in limiting angles $2\pi/3$ and $- 2\pi/3$ for positive and negative coefficients B_3, respectively. A stronger condition for α can be obtained by excluding the appearance of two zero points of the velocity component U near the edge. Then, the angle between plate and streamline at the edge must be equal or smaller than $\frac{\pi}{2}$.

The described type of separation at the leading edge can be observed on photographs [8, page 683]. However, there results an important consequence for the ideal flow theory which will be explained briefly.

In ideal flow theory vorticity is introduced either in form of discrete vortices or in form of continuous vortex sheets in order to remove the singularity in the velocity at the edge of a body. The strengths of these vortices or the vortex density of the sheets are determined by the well-known Kutta condition which requires a finite velocity at the edge (see, for instance [1]). This condition implies a tangential streamline at the edge. Also Kirchhoff's discontinuity lines (free streamlines) are tangential at the edge (see, for instance [2]). These propositions are obviously in contradiction to the real flow behavior, at least for one-phase flows.* It is interesting that Rott [4] already questioned the validity of the Kutta condition for a single vortex model. He found an angle of separation of exactly $\frac{\pi}{2}$ on the basis of ideal flow theory by abandoning the Kutta condition.

Equation (50) offers besides $\alpha \neq 0$, $B_2 = 0$ also the alternative $\alpha = 0$, $B_2 \neq 0$. The equations (51), (52), and (53) yield

$$B_3 = 0 \tag{58}$$

$$\beta_1 = 0, \; \beta_2 = -2\frac{A_3}{B_2} \tag{59}$$

$$\gamma_1 = -\frac{B_5}{7A_3}, \; \gamma_2 = +\frac{B_5}{7A_3} \tag{60}$$

respectively.

* In cavities the assumption of tangential discontinuity lines is realistic since the tangential velocity component dominates at the edge in the presence of a two-phase boundary.

For non-vanishing coefficients A_3 and B_2 two streamlines originate at the edge under 180° with curvatures which can be expressed by the coefficients $\beta_1 = 0$ and $\beta_2 = -2\frac{A_3}{B_2}$. An inspection of the flow patterns by means of the velocity components U and V reveals that only the class

$$A_3 < 0 \ , \quad B_2B_5 > 0 \tag{61}$$

is of interest for the present discussion (Figure 4). This class may be called "separated flows around the trailing edge of a plate."

The shear stress at the surface is now (because of $B_2 \neq 0$) of Couette type. The pressure drop is the same as for the separated flows around the leading edge.

The separated flow described near the edge (Figure 4) can be part of a flow field which is produced by a source located at the upper right side of the semi-infinite plate and which is sketched in Figure 5. This indicates the occurrence of "secondary" vortices near the edge. Rott [4] has pointed to photographs which clearly show the existence of secondary vortices despite their tiny extension (see, for instance, the flow pattern near a wedge by Prandtl in [3]).

Here again the propositions of ideal flow theory must be reexamined. One can immediately see that a single vortex model or a vortex sheet, which is either continuous or composed of discrete vortices, are not an adequate means to describe this type of separation.

It remains to check the existence of a streamline at the edge which justifies the introduction of the quasi-symmetric flows. Adopting the conditions $A_3 = B_2 = \alpha = 0$ from the symmetric trivially separated flows one finds with the equations (51) through (53):

$$B_3 = 0 \tag{65}$$

$$B_2\beta^2 = 0 \tag{66}$$

$$B_5 = 0 \tag{67}$$

and from the equations of the next higher orders

$$A_5\beta = 0 \tag{68}$$

$$3A_4\beta^2(\delta + \beta) + 9A_5\gamma + B_7 = 0 \tag{69}$$

Since $A_5 \neq 0$ there follows $\beta = o$ and

$$\gamma = -\frac{B_7}{A_5} \tag{70}$$

The coefficient B_7, therefore, causes the slight nonsymmetry of the quasi-symmetric flow as was anticipated.

It may be mentioned that the double series (30), (31), and (32) for the velocity components U, V and the pressure P permit the existence of other classes of either attached or separated flows past the edge of a plate. For instance, the lowest order terms might vanish and higher order terms become of leading importance.

Also, if

$$A_3 > 0 \ , \quad B_2B_3 < 0 \tag{71}$$

the differential equation for the streamlines (41) yields a solution of the form

$$\xi = \pm \sqrt{\frac{2}{5}\frac{B_3}{B_2}}\ \eta^{\frac{3}{2}} + \ldots \tag{72}$$

which is not analytic at the origin $\eta = 0$ as is assumed in equation (49). However, a simple examination of the flow directions reveals that these flows are of only remote practical interest and may, therefore, be omitted in this discussion.

Finally, it may be mentioned that an introduction of polar coordinates around the edge of the plate would have led to exactly the same results. This method will be presented in a forthcoming paper which deals with the more general wedge and corner problems that necessitate the introduction of polar coordinates.

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APPENDIX A

THE LEADING REDUCED EQUATIONS OF MOTION

a. The reduced equations of continuity:

$$n = 0: \quad V_1 - U_0' = 0 \tag{A - 1}$$

$$n = 1: \quad U_1 + 2\eta V_2 - \eta U_1' + V_0' = 0 \tag{A - 2}$$

$$n = 2: \quad 2U_2 + 3\eta V_3 - \eta U_2' + V_1' = 0 \tag{A - 3}$$

$$n = 3: \quad 3U_3 + 4\eta V_4 - \eta U_3' + V_2' = 0 \tag{A - 4}$$

$$n = 4: \quad 4U_4 + 5\eta V_5 - \eta U_4' + V_3' = 0 \tag{A - 5}$$

$$n = 5: \quad 5U_5 + 6\eta V_6 - \eta U_5' + V_4' = 0 \tag{A - 6}$$

$$n = 6: \quad 6U_6 + 7\eta V_7 - \eta U_6' + V_5' = 0 \tag{A - 7}$$

b. The reduced first Navier-Stokes equations:

$$n = 0: \quad U_0'' + 2U_2 - \eta P_0' = 2\eta V_0 U_1 - 2\eta U_0 U_0' \tag{A - 8}$$

$$\begin{aligned} n = 1: \quad & U_1'' + 6U_3 - \eta P_1' + P_1 = \\ & 2U_0U_1 + 4\eta V_0U_2 - 2\eta U_0U_1' + 2V_0U_0' \end{aligned} \tag{A - 9}$$

$$\begin{aligned} n = 2: \quad & U_2'' + 12U_4 - \eta P_2' + 2P_2 = \\ & 2U_1^2 + 2\eta U_1V_2 + 4U_0U_2 + 2\eta U_0'U_2 + 6\eta U_3V_0 \\ & + 2U_1'V_0 - 2\eta U_1'U_1 + 2U_0'^2 - 2\eta U_2'U_0 \end{aligned} \tag{A - 10}$$

$$\begin{aligned} n = 3: \quad & U_3'' + 20U_5 - \eta P_3' + 3P_3 = \\ & 6U_1U_2 + 2\eta U_1V_3 + 4\eta U_2V_2 + 6U_0U_3 + 4\eta U_3U_0' \\ & - 8\eta U_4V_0 + 2U_0'V_2 + 2U_1'U_0' - 2\eta U_1'U_2 + 2U_2'V_0 \\ & - 2\eta U_2'U_1 - 2\eta U_3'U_0 \end{aligned} \tag{A - 11}$$

n = 4 : $U_4'' + 30U_6 - \eta P_4' + 4P_4 =$

$8U_1U_3 + 2\eta U_1U_4 + 4U_2^2 + 4\eta U_2V_3 + 6\eta U_3V_2 + 8U_0U_4$

$+ 6\eta U_4U_0' + 10\eta U_5V_0 + 2U_0'V_3 + 2V_2U_1' - 2\eta U_3U_1'$

$+ 2U_0'U_2' - 2\eta U_2U_2' + 2V_0U_3' - 2\eta U_1U_3' - 2\eta U_0U_4'$ (A - 12)

n = 5 : $U_5'' + 42U_7 - \eta P_5' + 5P_5 =$

$2V_4U_0' + 8\eta U_5U_0' + 2V_3U_1' - 2\eta U_4U_1' + 2U_4U_1$

$+ 2\eta V_5U_0' + 2V_2U_2' - 2\eta U_3U_2' + 4U_3U_2$

$+ 4\eta V_4U_2 + 2U_0'U_3' - 2\eta U_2U_3' + 6U_2U_3 + 6\eta V_3U_3$

$+ 2V_0U_4' - 2\eta U_1U_4' + 8U_1U_4 + 8\eta V_2U_4 - 2\eta U_0U_5'$

$+ 10U_0U_5 + 12\eta V_0U_6$ (A - 13)

c. <u>The reduced second Navier-Stokes equations:</u>

n = 0 : $V_0'' + 2V_2 + \eta P_1 = 2\eta V_0U_0' - 2\eta U_0V_0'$ (A - 14)

n = 1 : $U_0''' + 6V_3 + 2\eta P_2 + P_0' =$

$2U_0'U_0 + 4\eta V_0V_2 - 2\eta U_0U_0'' + 2\eta U_0'^2$

$+ 2V_0'V_0 - 2\eta V_0'U_1$ (A - 15)

n = 2 : $V_2'' + 12V_4 + 3\eta P_3 + P_1' =$

$2U_0'U_1 + 6\eta U_0'V_2 + 4U_0V_2 - 6\eta V_0V_3 + 2U_0'V_0'$

$- 2\eta V_0'U_2 + 2V_0U_0'' - 2\eta U_0''U_1 - 2\eta U_0V_2'$ (A - 16)

n = 3 : $V_3'' + 20V_5 + 4\eta P_4 + P_2' =$

$2U_0'U_2 + 8\eta U_0'V_3 + 4U_1V_2 + 4\eta V_2^2 + 6U_0V_3$

$- 8\eta V_0V_4 + 2V_0'V_2 - 2\eta V_0'U_3 + 2U_0''U_0' - 2\eta U_0''U_2$

$+ 2V_0V_2' - 2\eta U_1V_2' - 2\eta U_0V_3'$ (A - 17)

$$n = 4: \quad V_4'' + 30V_6 + 5\eta P_5 + P_3' +$$

$$2U_3U_0' + 10\eta U_0'V_4 + 4U_2V_2 + 10\eta V_2V_3$$

$$+ 6U_1V_3 + 8U_0V_4 + 10\eta V_0V_5 + 2V_3V_0'$$

$$- 2\eta U_4V_0' + 2V_2U_0'' - 2\eta U_3U_0'' + 2U_0'V_2'$$

$$- 2\eta U_2V_2' + 2V_0V_3' - 2\eta U_1V_3' - 2\eta U_0V_4' \qquad (A - 18)$$

$$n = 5: \quad V_5'' + 42V_7 + 6\eta P_6 + P_4' =$$

$$2V_4V_0' - 2\eta U_5V_0' + 2U_4U_0' + 12\eta V_5U_0'$$

$$+ 2V_3U_0'' - 2\eta U_4U_0'' + 4U_3V_2 + 4\eta V_2V_4$$

$$+ 2V_2V_2' - 2\eta U_3V_2' + 6U_2V_3 + 6\eta V_3^2$$

$$+ 2U_0'V_3' - 2\eta U_2V_3' + 8U_1V_4 + 8\eta V_2V_4$$

$$+ 2V_0V_4' - 2\eta U_1V_4' + 10U_0V_5 - 2\eta U_0V_5' + 12\eta V_0V_6 \qquad (A - 19)$$

The equations (A - 8) through (A - 19) are simplified by considering equation (A - 1).

APPENDIX B

THE FIRST SEVEN SUBSETS OF ALGEBRAIC EQUATIONS.

First subset:

Vanishing coefficients because of conditions (16) and (18).

Second subset:

From equations

(A - 1), η^1 : $U_{02} = 0$

(A - 2), η^1 : $U_{11} - U_{11} = 0$

(A - 9), η^0 : $V_{02} = 0$

Third subset:

From equations

(A - 2), η^2 : $3B_3 - U_{12} = 0$

(A - 3), η^1 : $U_{21} + 6A_3 = 0$

(A - 8), η^1 : $2U_{21} + 6A_3 - P_{01} = 0$

(A - 14), η^1 : $6B_3 + P_{10} = 0$

(A - 9), η^0 : $2U_{12} + P_{10} = 0$

(A - 15), η^0 : $P_{01} + 2V_{12} = 0$

Fourth subset:

(A - 2), η^3: $U_{13} + 4B_4 + 2V_{22} - 3U_{13} = 0$

(A - 8), η^2: $- 2P_{02} + 2U_{22} + 12A_4 = 0$

(A - 14), η^2: $P_{11} + 2V_{22} + 12B_4 = 0$

(A - 3), η^2: $A_4 = 0$

(A - 9), π^1 : $U_{31} + U_{13} = 0$

(A - 15), π^1 : $2P_{20} + 2P_{02} + 6V_{13} = 0$

(A - 4), π^1 : $U_{31} + V_{22} = 0$

(A - 10), π^0 : $2P_{20} + 2U_{22} = 0$

(A - 16), π^0 : $P_{11} + 2V_{22} = 0$

Fifth Subset

(A - 2), π^4 : $5B_5 + 2V_{23} - 3U_{14} = 0$

(A - 8), π^3 : $- 3P_{03} + 2U_{23} + 20A_5 = 0$

(A - 14), π^3 : $P_{12} + 2V_{23} + 20B_5 = 0$

(A - 3), π^3 : $20A_5 - U_{23} + 3V_{32} = 0$

(A - 9), π^2 : $- P_{12} + 6U_{32} + 12U_{14} = 0$

(A - 15), π^2 : $- 2P_{21} + 3P_{03} + 6V_{32} + 12V_{14} = 0$

(A - 4), π^2 : $U_{32} + 3V_{23} = 0$

(A - 10), π^1 : $P_{21} + 12U_{41} + 6U_{23} = 0$

(A - 16), π^1 : $3P_{30} + 2P_{12} + 6V_{23} = 0$

(A - 5), π^1 : $3U_{41} + 2V_{32} = 0$

(A - 11), π^0 : $3P_{30} + 2U_{32} = 0$

(A - 17), π^0 : $P_{21} + 2V_{32} = 0$

Sixth Subset

(A - 1), π^5 : $V_{15} - 6A_6 = 0$

(A - 2), π^5 : $6B_6 + 2V_{24} - 4U_{15} = 0$

(A - 8), π^4 : $- 4P_{04} + 2U_{24} + 30A_6 = 0$

(A - 14), π^4 : $P_{13} + 2V_{24} + 30B_6 = 0$

(A - 3), π^4 : $30A_6 + 3V_{33} - 2U_{24} = 0$

(A - 9), π^3 : $-2P_{13} + 6U_{33} + 20U_{15} = 0$

(A - 15), π^3 : $2P_{22} + 4P_{04} + 6V_{33} + 20V_{15} = 0$

(A - 4), π^3 : $V_{24} + V_{42} = 0$

(A - 10), π^2 : $U_{24} + U_{42} = 0$

(A - 16), π^2 : $P_{31} + P_{13} + 4V_{42} + 4V_{24} = 0$

(A - 5), π^2 : $4U_{42} + 3V_{33} - 2U_{42} = 0$

(A - 11), π^1 : $2P_{31} + 20U_{51} + 6U_{33} = 0$

(A - 17), π^1 : $4P_{40} + 2P_{22} + 6V_{33} = 0$

(A - 6), π^1 : $2U_{51} + V_{42} = 0$

(A - 12), π^0 : $2P_{40} + U_{42} = 0$

(A - 18), π^0 : $P_{31} + 2V_{42} = 0$

<u>Seventh Subset</u>

(A - 1), π^6 : $7A_7 - V_{16} = 0$

(A - 2), π^6 : $-5U_{16} + 7B_7 + 2V_{25} = 0$

(A - 8), π^5 : $-5P_{05} + 2U_{25} + 42A_7 = 2B_3U_{11}$

(A - 14), π^5 : $P_{14} + 2V_{25} + 42B_7 = 0$

(A - 3), π^5 : $3V_{34} - 3U_{25} + 42A_7 = 0$

(A - 9), π^4 : $-P_{14} + 2U_{34} + 10U_{16} = 0$

(A - 15), π^4 : $2P_{23} + 5P_{05} + 6V_{34} + 30V_{16} = -6B_3U_{11}$

(A - 4), π^4 : $5V_{25} + 4V_{43} - U_{34} = 0$

(A - 10), π^3 : $-P_{23} + 12U_{43} + 20U_{25} = 2B_3U_{11} - 2U_{12}U_{11}$

(A - 16),	η^3 :	$3P_{32} + 4P_{14} + 12V_{43} + 20V_{25} = 6A_3U_{11} - 4V_{12}U_{11}$
(A - 5),	η^3 :	$U_{43} + 4V_{34} + 5V_{52} = 0$
(A - 11),	η^2 :	$P_{32} + 20U_{52} + 12U_{34} = 2U_{21}U_{11} + 2V_{12}U_{11}$
(A - 17),	η^2 :	$4P_{41} + 3P_{23} + 20V_{52} + 12V_{34} = 0$
(A - 6),	η^2 :	$U_{52} + V_{43} = 0$
(A - 12),	η^1 :	$P_{41} + 10U_{61} + 2U_{43} = 0$
(A - 18),	η^1 :	$5P_{50} + 2P_{32} + 6V_{43} = 0$
(A - 7),	η^1 :	$5U_{61} + 2V_{52} = 0$
(A - 13),	η^0 :	$5P_{50} + 2U_{52} = 0$
(A - 19),	η^0 :	$P_{41} + 2V_{52} = 0$

APPENDIX C

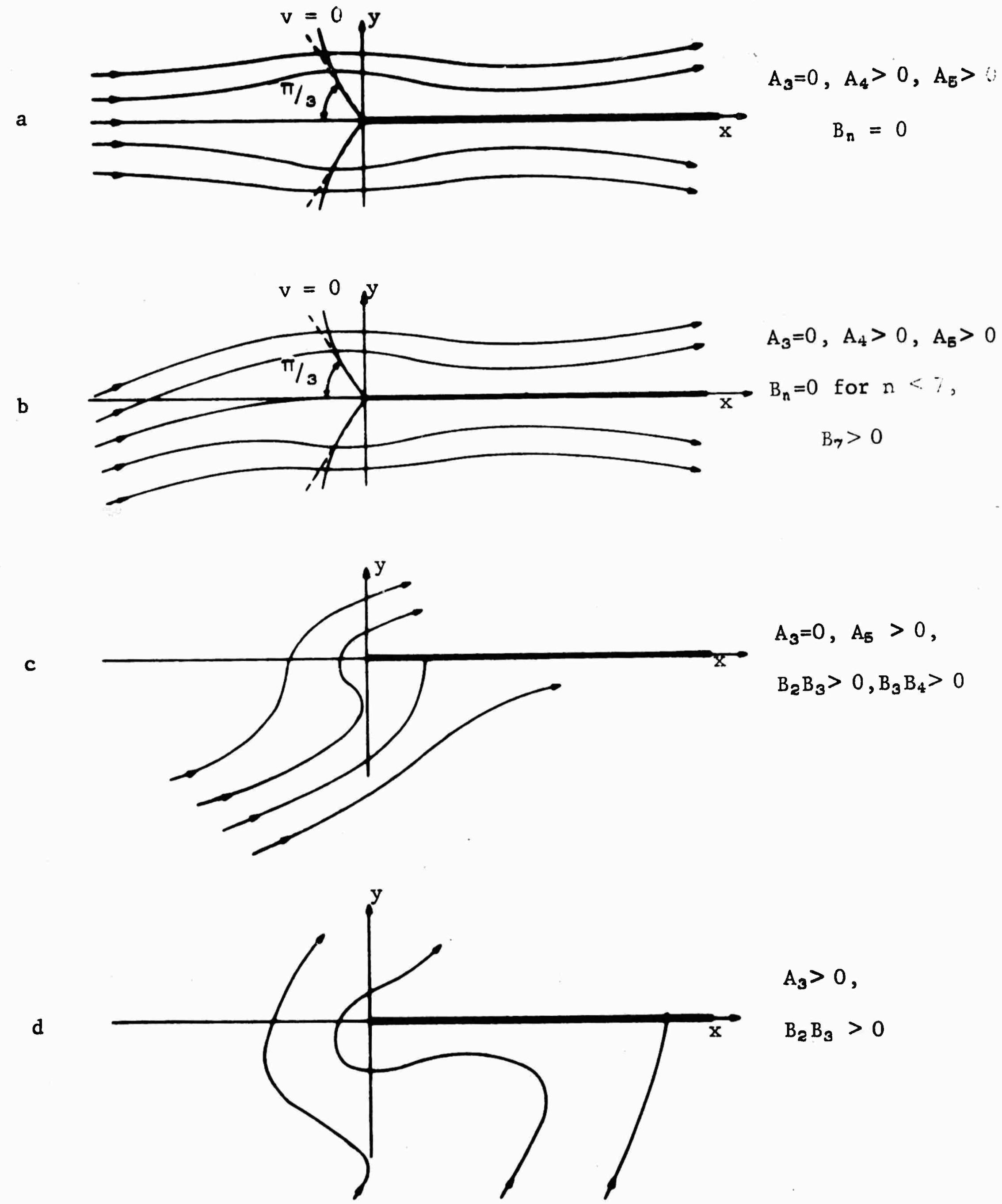

FIGURE 1: Flow patterns of laminar motions around the leading edge of a semi-infinite flat plate for different angles of attack.

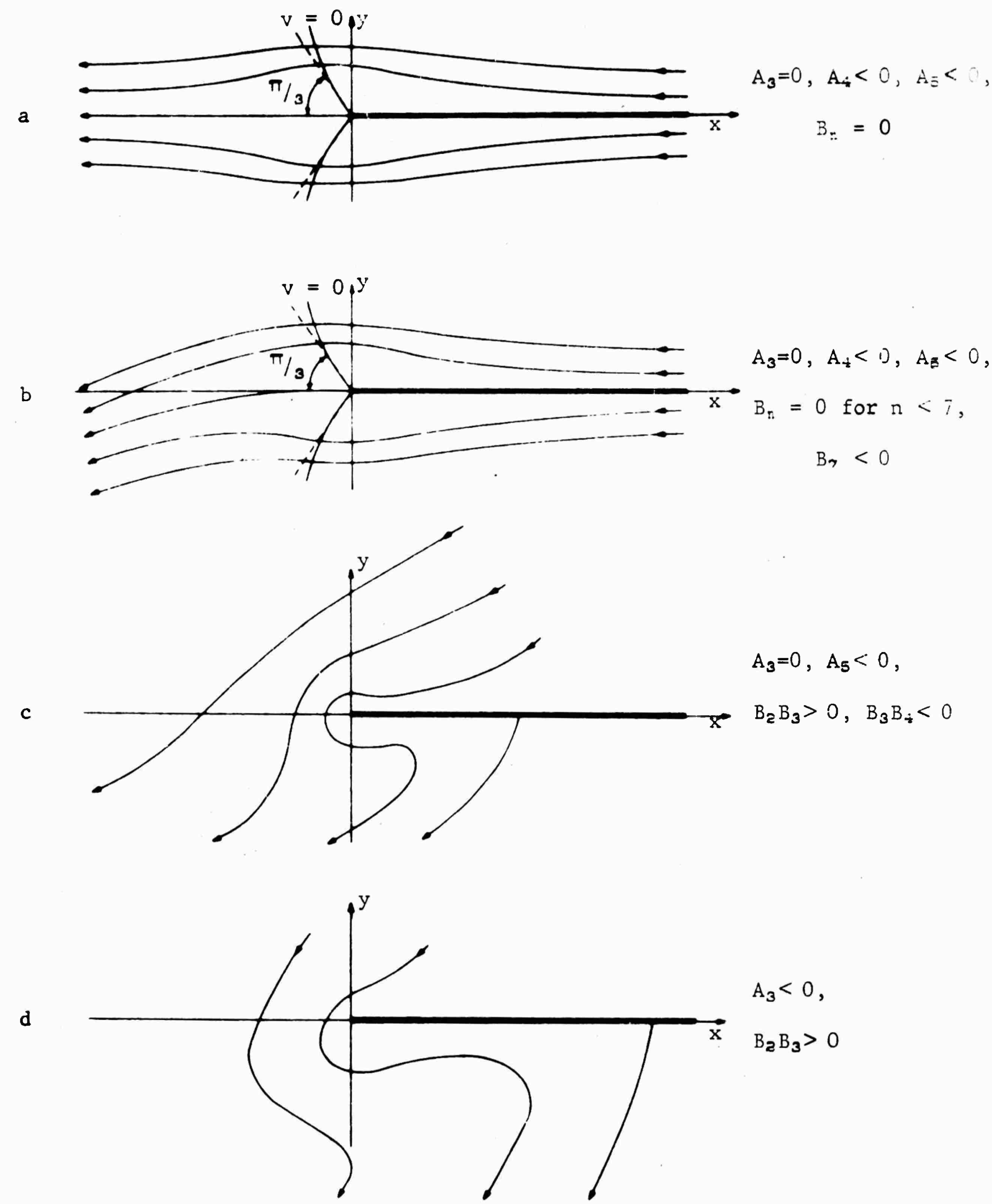

FIGURE 2: Flow patterns of laminar motions around the trailing edge of a semi-infinite flat plate for different angles of attack.

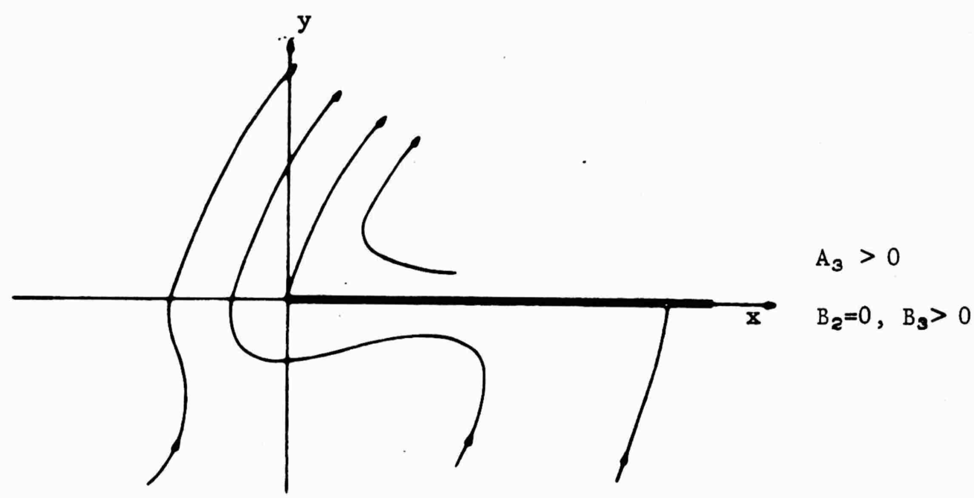

FIGURE 3: Separated flow around the leading edge of a semi-infinite flat plate.

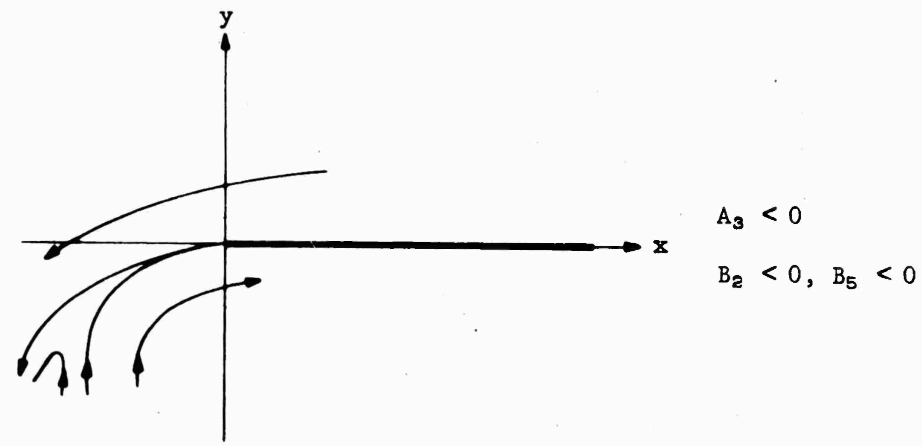

FIGURE 4: Separated flow around the trailing edge of a semi-infinite flat plate.

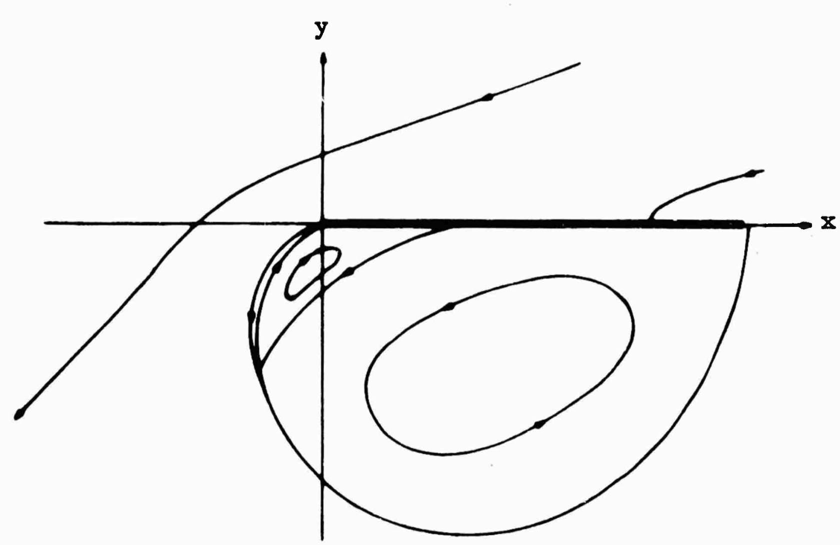

FIGURE 5 : Sketch of a separated flow around the trailing edge of a semi-infinite flat plate with a source on the upper right side.

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Oct 1963. 30 p. 5 figs.
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The characteristics of general laminar flows around the edge of a semi-infinite flat plate are investigated on the basis of the complete Navier-Stokes equations. The solution of this singular and non-linear problem is obtained in an exact and linear manner in the form of a double series around the edge. It permits a rigorous discussion of the properties of flows around an edge.

1. Laminar flow - Properties
2. Navier-Stokes equations
3. Meteorology
I. Lugt, H. J.
II. Schwiderski, E. W.
III. Title

Task: R360FR103/2101/ R01101001

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REPORT NUMBER 1887	1887	CIRCULATION LIMITATION	
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SUBJECT ANALYSIS OF REPORT

DESCRIPTOR	CODE	DESCRIPTOR	CODE	DESCRIPTOR	CODE
Laminar	LAMI	Fluids	FLUI		
Flow	FLOW	Motion	MOTI		
Leading	LEAI	Theory	THEY		
Edges	EDGE	Kutta (Name)	KUTT		
Trailing Edge	TRAG	Reliability	RELI		
Semi	SMIZ	Mathematics	MATH		
Infinite	IFIT	Vortex	VORT		
Plates	PLAT	Symmetrical	SYMM		
Aerodynamics	AERD	Non-symmetrical	SYMMX		
Navier-Stokes	NAVE	Equations of motion	EQUM		
Equations	EQUA	Meteorology	METE		
Properties	PROA	Boundary layer	BOUL		
Viscosity	VISC	Wakes	WAKE		
Non-viscous	VISOX				